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Problem. Prove the Pinching Theorem: *Suppose that for all sufficiently large n,*

$$a_n \leq b_n \leq c_n.$$

If $\lim a_n = \lim c_n = L$, *then* $\lim b_n = L$.

Solution. To prove this, we will use the definition of the limit of a sequence. Because we know that $\lim a_n = \lim c_n = L$, we know by the definition of the limit of a sequence that,

$$L - \epsilon_a < a_n < L + \epsilon_a$$

for any ϵ_a with all sufficiently large n and that,

$$L - \epsilon_c < c_n < L + \epsilon_c$$

for any ϵ_c with all sufficiently large n.

Because we know that $a_n < c_n$, let $\epsilon = max\{\epsilon_a, \epsilon_c\}$. Now we get

$$L - \epsilon < a_n \leq c_n < L + \epsilon$$

But we know that $a_n \leq b_n \leq c_n$. For any ϵ, then there exists an ϵ_b such that for sufficiently large n,

$$L - \epsilon \leq L - \epsilon_b < a_n \leq b_n \leq c_n < L + \epsilon_b \leq L + \epsilon.$$

Removing the unneccesary terms of this inequality we get,

$$L - \epsilon < b_n < L + \epsilon$$

for any ϵ with all sufficiently large n. This is equivalent to saying that $\lim b_n = L$. $\square$